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Activity 4-1 (20 Jul 2021)

1. (source: LPV) Prove by induction on *k* that for any integer $k \geq 1$, we have that

$$1+3+\cdots+(2k-1)=k^2$$

State the property *P*(*k*):

P(*k*)	

Basic step: *(show that P(1) is true)*

Induction step: (assume *P*(*m*) and show *P*(*m*+1), for any *m*>=1)

State the Induction Hypothesis *P*(*m*):

P(*m*)	

State the goal *P*(*m*+1)

P(*m+1*)	

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Activity 4-2 (20 Jul 2021)

2 (source: MN) Prove that for any integer $n \geq 0$, the following formula is true:

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1 \quad .$$

State the property $P(n)$:

$P(n)$	

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Activity 4-3 (20 Jul 2021)

In this problem, we will try to prove that there is an arbitrary large gap between two primes. (This is the statement in the video clip quiz.) More specifically, prove that for any positive integer L, there is a sequence of L consecutive integers all consisting of composite integers.

(*Hint*: You should prove by construction. Let $n = L+1$. Consider $n! + 2$. What can you say about it? How about $n! + 3$?)

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Activity 4-4 (20 Jul 2021)

4. (R-3.3-ex-12) Prove that $3^n < n!$ whenever n is a positive integer greater than 6.

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Activity 4-5 (20 Jul 2021)

5. (LPV-2.1.5) Prove the following identity:

$$1\cdot 2+2\cdot 3+3\cdot 4+\cdots+(n-1)\cdot n=\frac{(n-1)\cdot n\cdot(n+1)}{3}$$