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**Activity 5-1 (29 Jul 2021)**

1. (MN-ex-1b) Prove that for integer  $n \geq 1$  ,

$$\sum_{i=1}^n i \cdot 2^i = (n-1)2^{n+1} + 2 .$$

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**Activity 5-2 (29 Jul 2021)**

2. (LPV-2.5.4b) Prove that for any integer  $n \geq 1$ ,  $n^3 - n$  is a multiple of 6.

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**Activity 5-3 (29 Jul 2021)**

3. (R-3.3-ex-37) Show that using only 3-baht and 5-baht coins, one can form a set of coins worth  $n$  baht for any integer  $n > 7$ .

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**Activity 5-4 (29 Jul 2021)**

4 The new kWatch smartwatch comes in many configurations. You can choose the face size of 1 inch or 2 inches. For the case, there are 3 options: steel, aluminium and gold. For the watch band, there are 20 options. In how many ways can you configure your new kWatch?

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**Activity 5-5 (29 Jul 2021)**

5 (a) (LPV1.5.5) There are 20 different presents. We want to give all of them to 12 children. Each children can get any number of presents (maybe 0), and we may give all presents to one child. In how many ways can we distribute these presents?

5 (b) (LPV1.5.6) There are 20 flavors of ice cream. There are 12 children. Each child can have as many ice cream as she or he wants, but she or he cannot have more than one scoop of each flavor. It is possible that some child does not want any ice cream. In this problem, the order that a child has an ice cream does not matter, i.e., a having a scoop of chocolate and a scoop of vanilla ice cream is the same as having a scoop of vanilla and a scoop of chocolate ice cream. In how many ways can these children have ice cream?

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**Activity 5-6 (29 Jul 2021)**

6. (R-3.3-ex-37) Show that if  $n$  is a positive integer then

$$\sum_{\{a_1, a_2, \dots, a_k\} \subseteq \{1, 2, \dots, n\}} \frac{1}{a_1 a_2 \cdots a_k} = n \quad .$$

In this problem, the sum is over all non-empty subsets of  $\{1, 2, \dots, n\}$ .

(Hint: prove by induction. Consider the sum for subsets containing  $k+1$  and for subsets not containing  $k+1$ .)