Activity 5-1 (2 Aug 2022)

1. (MN-ex-1b) Prove that for integer $n \geq 1$,

$$\sum_{i=1}^{n} i \cdot 2^i = (n-1)2^{n+1} + 2 \quad .$$

Activity 5-2 (2 Aug 2022)

2. (LPV-2.5.4b) Prove that for any integer $n \geq 1$, $n^3 - n$ is a multiple of 6.

Activity 5-3 (2 Aug 2022)

3. (R-3.3-ex-37) Show that using only 3-baht and 5-baht coins, one can form a set of coins worth n baht for any integer $n > 7$.

Activity 5-4 (2 Aug 2021)

4. (R-3.3-ex-37) Show that if n is a positive integer then

$$\sum_{\{a_1, a_2, \ldots, a_k\} \subseteq \{1,2,\ldots,n\}} \frac{1}{a_1 a_2 \cdots a_k} = n \ .$$

In this problem, the sum is over all non-empty subsets of $\{1,2,\ldots,n\}$.

(Hint: prove by induction. Consider the sum for subsets containing $k+1$ and for subsets not containing $k+1$.)