01204211: Exercises 10-1

Notes: To avoid confusion, in your answer, you can write vectors with this notation $\vec{u}$.

1. In this problem, we will use Gaussian Elimination to solve the following linear system.

$$\begin{array}{rcrcrcl} x_1 & + & 3x_2 & + & 2x_3 & = & 7 \\ 5x_1 & + & 10x_2 & + & x_3 & = & 8 \\ 2x_1 & - & 4x_2 & - & 4x_3 & = & 9 \end{array}$$

   (a) Formulate the linear system in matrix form. Denote the coefficient matrix by A.

   (b) Use elimination process to transform A into an upper triangular matrix. Write down every step. (There should be 3 steps.)

   Also, for each step i, (1) write down an elementary matrix E_i that represents the elimination process that you perform and (2) write down its inverse E_i^{-1}.

(c) Call the resulting upper triangular matrix U. Write down, in full, the matrix equation $E_3E_2E_1A = U$.

(d) Use the inverses found in question (b) to write down another equation that shows how to perform matrix multiplication to obtain A from U, i.e., $A = E_1^{-1}E_2^{-1}E_3^{-1}U$.

(e) Let $L = E_1^{-1}E_2^{-1}E_3^{-1}$. Find L and write down the LU decomposition of A, i.e., $A = LU$.

(f) Consider the linear system $Ax = b$ when $A = LU$. Write down the starting linear system that we want to solve in the matrix form, but this time replace A with LU. Explain briefly what the two matrices U and L "do" to x to obtain b.

$$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$