01204213: Homework 2

Due: 23pm, 19 Jul 2021.

1. (Siper 1.6.cont) Give state diagrams of finite automata recognizing the following languages. In all parts the alphabet is $\{0, 1\}$.

   (a) $\{w \mid w$ contains the substring 0101, i.e., $w = x0101y$ for some x and $y\}$

   (b) $\{w \mid w$ is any string except 11 and 111$\}$

   (c) $\{w \mid$ every odd position of w is a 1$\}$

   (d) $\{w \mid$ the length of w is at most 5$\}$

2. (Sipser 1.8) Use the construction described in class to give an NFA recognizing the union of languages in problem 1(a) and 1(c).

3. (Sipser 1.9) Use the construction described in class to give an NFA recognizing the concatenation of languages in problem 1(d) and 1(b).

4. (Sipser 1.10) Use the construction described in class to give an NFA recognizing the star of language in problem 1(c).

5. (Sipser 1.14)

   (a) Show that if M is a DFA that recognizes language B, swapping the accept and nonaccept states in M yields a new DFA recognizing the complement of B. Conclude that the class of regular languages is closed under complement.

   (b) Show by giving an example that if M is an NFA that recognizes language C, swapping the accept and nonaccept states in M doesn't necessarily yield a new NFA that recognizes the complement of C. Is the class of languages recognized by NFAs closed under complement? Explain your answer.

6. (Sipser 1.17)

   (a) Give an NFA recognizing the language $(\mathtt{01} \cup \mathtt{001} \cup \mathtt{010})^*$

   (b) Convert this NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the start state.

7. (Sipser 1.19) Use the procedure described in lecture to convert the following regular expression to nondeterministic finite automata.

   (a) $(\mathtt{0} \cup \mathtt{1})^*\mathtt{000}(\mathtt{0} \cup \mathtt{1})^*$

   (b) $(((\mathtt{00})^*(\mathtt{11})) \cup \mathtt{01})^*$

   (c) $\emptyset^*$

8. (Sipser 1.41) Let $B_n = \{a^k \mid k$ is multiple of $n\}$. Show that the language B_n is regular.

9. (Sipser 1.31) For languages A and B, let the *perfect shuffle* of A and B be the language

   $$\{w \mid w = a_1 b_1 a_2 b_2 \cdots a_k b_k \text{ where } a_1 a_2 \cdots a_k \in A \text{ and } b_1 b_2 \cdots b_k \in B, \text{ each } a_i, b_i \in \Sigma\}\}$$

   Show that the class of regular languages is closed under perfect shuffle.