

# 01204213: Homework 7

Due: 23pm, 20 Sep 2021.

1. (Sipser 5.1) In this problem we consider problems related to context free grammars. Let

$$ALL_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*,\}$$

be the language of all CFGs that accepts all strings. Also let

$$EQ_{CFG} = \{\langle G_1, G_2 \rangle \mid G_1 \text{ and } G_2 \text{ are CFGs and } L(G_1) = L(G_2),\}$$

be the language of equivalent CFGs.

It is proved in the book that  $ALL_{CFG}$  is undecidable. Use this fact to prove that  $EQ_{CFG}$  is undecidable.

2. (Sipser 5.2) Show that  $EQ_{CFG}$  is co-Turing recognizable.
3. (Sipser 5.9) Let  $T = \{\langle M \rangle \mid M \text{ is a TM that accepts } w^{\mathcal{R}} \text{ whenever it accepts } w\}$ . Show that  $T$  is undecidable.
4. (Sipser 5.23) Show that  $A$  is decidable iff  $A \leq_m 0^*1^*$ .
5. (Sipser 5.22) Show that  $A$  is Turing-recognizable iff  $A \leq_m A_{TM}$ .

Hint: Recall that a language  $A$  is Turing-recognizable iff there exists a TM  $M$  such that for every string  $w$ ,  $M$  accepts  $w$  iff  $w \in A$ . You can use this definition in your solution. More specifically, you may want to use the fact that there exists a TM  $M$  in this definition.