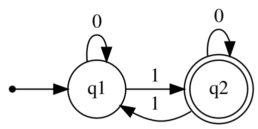
01204213: Homework 1

Due: 23pm, 20 Jul 2022.

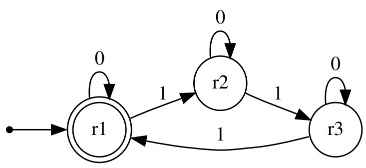
1. (Siper 1.6) Give state diagrams of finite automata recognizing the following languages. In all parts the alphabet is $\{0, 1\}$.
   (a) $\{w \mid w$ begins with a 1 and ends with 0$\}$
   (b) $\{w \mid w$ contains at least three 1's$\}$
   (c) $\{w \mid w$ starts with 0 and has odd length, or starts with 1 and has even length$\}$
   (d) $\{\varepsilon, 0\}$
   (e) All strings except the empty string

2. (Sipser 1.12) Let $\{\texttt{a}, \texttt{b}\}$ denote the alphabet. Let $D = \{w \mid\ w$ contains an even number of `a`'s and and odd number of `b`'s and does not contain the substring `ab` $\}$. Give a finite automaton with 5 states that recognizes D. (Suggestion: Describe D more simply.)

3. Consider the following finite automata M_1 and M_2.



M_1:



M_2:

   (a) What language does M_1 recognize?
   (b) What language does M_2 recognize?
   (c) For $i \in \{1, 2\}$, let A_i denote the language recognized by M_i. Use the construction we discussed in class to construct a finite automaton M that recognizes $A_1 \cup A_2$.

4. Let A_1 and A_2 be regular languages. Prove that $A_1 \cap A_2$ is also a regular language.

5. (Sipser 1.36) For any string $w = w_1 w_2 \cdots w_n$, the *reverse of* w, written $w^{\mathcal{R}}$, is the string w in reverse order, $w_n w_{n-1} \cdots w_2 w_1$. For any language A, let $A^{\mathcal{R}} = \{w^{\mathcal{R}} | w \in A\}$. Prove that if A is regular, so is $A^{\mathcal{R}}$.

   (Hint: First, try to prove the case when the finite automaton recognizing A has only one accept state. Then, using the result proved in class (the union of regular languages is regular) to prove the required statement.)